

Island Cosmology in the Landscape

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In the eternally inflationary background driven by the metastable vacua of the landscape, it is possible that some local quantum fluctuations with the null energy condition violation can be large enough to stride over the barriers among different vacua, so that create some islands full of radiation in new vacua, and then these emergently thermalized islands will enter into the evolution of standard big bang cosmology. In this paper, we calculate the spectrum of curvature perturbation generated during the emergence of island. We find that generally the spectrum obtained is nearly scale invariant, which can be well related to that of slow roll inflation by a simple duality. This in some sense suggests a degeneracy between their scalar spectra. In addition, we also simply estimate the non-Gaussianity of perturbation, which is naturally large, yet, can lie in the observational bound well. The results shown here indicate that the island emergently thermalized in the landscape can be consistent with our observable universe.

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I. INTRODUCTION

In the eternally inflationary background driven by the metastable vacua of the landscape [1, 2], generally the bubbles with new vacua will nucleate in original vacua [3, 4], which are mediated by the CDL instanton [5]. However, these bubbles are either empty or dominated by the new vacua. Thus in order to obtain an observable universe, a slow roll inflation and subsequent reheating has to be required inside the bubble. However, recently, it has been argued that some local quantum fluctuations with the null energy condition violation might be large enough to stride over the barrier among different vacua in the landscape, so that straightly create many thermalized regions in new vacua, some of which may correspond to our observable universe [6]. The “thermalized” here means that the resulting state is a thermal state full of radiation and matter, in which all components are assumed to be in thermal equilibrium. Thus this part of region after the thermalization is quite similar to that after the reheating following a slow roll inflation, and so can be followed by a standard FRW evolution.

These thermalized regions are referred as the “islands” in Ref. [6], which in some sense inherits but is actually slightly different from the original idea of Ref. [7] and Ref. [8], since here these islands are suggested to origin from large fluctuations in parent vacuum but emerges in a different or baby vacuum. It is this difference that makes the emergence of the island here inevitably be related to the tunneling in the landscape, especially the HM tunneling [9]. We may phenomenally illustrate the island universe by applying the HM instanton. The HM instanton corresponds a fluctuation which makes the field jump to the top of the potential barrier, and then the field will rapidly roll down along the another side of barrier to the new vacuum and so it can be expected that the same reheating as that after slow roll inflation will occur, see Fig.1. In this sense, the emerging probability of the island in new vacua can be approximately given by that of the HM instanton between different vacua. There have

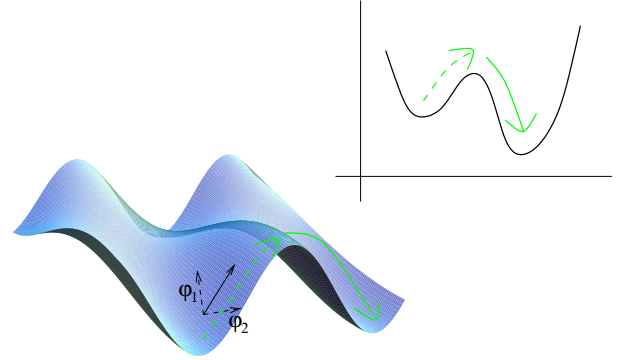


FIG. 1: The left down panel is the figure of a landscape with two scalar fields, in which an upward fluctuation will inevitably involve the fluctuations of both fields. To phenomenally simulate this kind of fluctuation, we need to introduce two fields with the reverse sign in their dynamical terms here. The right up panel is its section, which is used to illustrate the island universe model depicted here. The dashed line denotes an upward jump to the top of the potential barrier, which can be realized by the HM instanton or others, and then the field will rapidly roll down along the another side of barrier to new vacuum, see the solid line, and the same reheating as that after slow roll inflation will be expected to occur.

been lots of studies on the HM instanton, e.g. [10, 11] and recent [12]. While in Refs. [7, 8], it is not quite clear how to calculate the emerging probability of an island. The island universe should be distinguished from other models based on the upward fluctuations, for examples, the study of Ref. [13], in which the state after the fluctuation is assumed to be an observable universe with many structures, and also the recycling universe proposed in Ref. [14], see also Ref. [15] and [16], in which the state after the fluctuation is another inflation state, see Ref. [17] for more discussions on upward fluctuations.

The island in the landscape can share same remark-

where t is negative and runs up to 0, and n is a positive parameter dependent of time. We can obtain $\epsilon = -\frac{1}{n} + \frac{dn}{(nh)dt}$, which will be used in the calculations of primordial perturbations. In addition, we also assume $|\frac{1}{n}| \gg |\frac{dn}{(nh)dt}|$ in this paper, which implies $|\frac{dn}{h(dt)}| \ll 1$, otherwise the perturbation spectrum will be not nearly scale invariant. This means that the change of n in unite of Hubble time must be quite small. The more rapid the fluctuation is, in principle the stronger it can be [7], which in some sense is also a reflection of the uncertainty relation between the energy and time in quantum dynamics. Thus to make the fluctuate be strong enough to create the islands of our observable universe, $|\epsilon| \gg 1$ is required [6, 8]. Thus though during the fluctuation the change of h is drastic, the expansion of the scale factor is extremely slow, since we have $a \sim (-t)^{\frac{1}{|\epsilon|}}$ from Eq.(1) for n approximately constant.

The e-folding number of mode with some scale $\sim 1/k$ leaving the horizon before the thermalization can be defined as

$$\mathcal{N} = \ln \left(\frac{a_e h_e}{ah} \right), \quad (2)$$

where the subscript ‘e’ denotes the quantity evaluated at the time of the thermalization, and thus $k_e = a_e h_e$ is the last mode to be generated. When taking $ah = a_0 h_0$, where the subscript ‘0’ denotes the present time, we generally have $\mathcal{N} \sim 50$, which is required by observable cosmology. By using Eq.(2), and also note that the scale factor a is nearly unchanged here, we can obtain

$$\mathcal{N} \cong \ln \left(\frac{h_e}{h_i} \right) \cong \ln \left(\frac{T_e}{\Lambda_i} \right)^2,$$

where $\Lambda_i \simeq h_i^{1/2}$ is the energy scale of original or parent vacuum and $T_e \simeq h_e^{1/2}$ is the thermalized temperature after the fluctuation, which is the same as the reheating temperature after the inflation, and also the constant $G \equiv 1$ is set in this paper. When taking $\mathcal{N} \simeq 50$ and $T_e \sim 10^{15} \text{Gev}$, we have $\Lambda_i \sim \text{TeV}$. For a lower Λ_i , T_e may be taken smaller. Thus it seems that the above condition can be satisfied easily, which indicates that the e-folding number required to solve the horizon problem of standard cosmology may be always obtained by selecting a low parent vacuum. The reason is that the smaller the energy scale of parent vacuum is, the larger its Hubble scale is, thus the e-folding number, see Fig.2 for a further illustration. In principle, T_e should be lower than that the monopoles production needs, while higher than TeV. This can not only avoid the monopole problem afflicting the standard cosmology, but helps to provide a solution to the baryogenesis.

This can be also explained as follows. The scale of the null energy condition violating region is generally required to be larger than the Hubble scale of original vacuum [7], see also Refs. [22] and [23, 24]. This is also assured by the application of the HM instanton action,

since in which the region tunnelling to the top of the barrier corresponds to the Hubble scale of the original vacuum, which have been understood by using the stochastic approach to inflation [25, 26]. This sets the initial value of local evolution of a , and since it is nearly unchanged during the fluctuation, we can have $a_e \simeq a_i \simeq 1/h_i$, which means that the smaller h_i is, the larger the scale of local thermalized region after the fluctuation is, and thus the e-folding number.

III. SPECTRUM OF PRIMORDIAL PERTURBATION IN ISLAND

A. The calculations of curvature perturbation

In this section, we will study the primordial perturbations of island cosmology in the landscape, and, without losing generality, will discuss the case with the scalar fields φ_1 and φ_2 with the reverse sign in their dynamical terms, see Fig.1. In Ref. [27], the relevant calculations have been done for constant ϵ , however, here we will relax the assumption of constant ϵ and allow it change, which will make the results more flexible for matching to the observations. We assume that both fields are uncoupled and regard their potentials not around the top as

$$V(\varphi_1) = \frac{n_1(3n+1)}{8\pi} \exp \left(- \int \sqrt{\frac{16\pi}{n_1}} d\varphi_1 \right), \quad (3)$$

$$V(\varphi_2) = \frac{n_2(3n+1)}{8\pi} \exp \left(- \int \sqrt{\frac{16\pi}{n_2}} d\varphi_2 \right), \quad (4)$$

respectively, where the determination of prefactors has been showed in Ref. [27], and in general both n_1 and n_2 are positive parameters dependent of the time in different forms. Note that in Ref. [27], n_1 and n_2 were regarded as constants, thus in Eqs.(3) and (4) the integrals in the exponents can be integrated, which actually makes Eqs.(3) and (4) have more simple forms. Here in order to have a detailed compare of spectrum with the observation we regard n_1 and n_2 be changed, thus the integrals in the exponents must be reserved. However, to have a simple equation of entropy perturbation, n_1/n_2 is constant is assumed here, which means that their changes with the time are same, and also $n_1 + n_2 = n$, where n is given by Eq.(1). Note that $n \ll 1$, thus both $n_1, n_2 \ll 1$, which suggests that the potential of both fields are very steep in Eqs.(3) and (4). The fields φ_1 and φ_2 during their evolution will climb up along their potentials, which is determined by the property of such fields, see e.g. Refs. [28, 29]. Thus in this sense they can be suitable for simulating the emergence of island in the landscape, since the emergence of island here actually corresponds to an upward fluctuation along the potential in the landscape.

We can decompose both fields into the field φ along the field trajectory, and the field s orthogonal to the tra-

jectory by making a rotation in the field space

$$d\varphi = \frac{\sqrt{n_1}d\varphi_1 + \sqrt{n_2}d\varphi_2}{\sqrt{n}}, ds = \frac{\sqrt{n_2}d\varphi_1 - \sqrt{n_1}d\varphi_2}{\sqrt{n}},$$

as has been done in Ref. [30]. In this case, the total potential $V(\varphi, s)$, which is the sum of Eqs.(3) and (4), can be rewritten as $\tilde{V}(s)e^{-\int d\varphi\sqrt{\frac{16\pi}{n}}}$, where

$$\begin{aligned} \tilde{V}(s) = & \frac{n_1(3n+1)}{8\pi} \exp\left(-\int \sqrt{\frac{16n_2\pi}{n_1n}} ds\right) \\ & + \frac{n_2(3n+1)}{8\pi} \exp\left(\int \sqrt{\frac{16n_1\pi}{n_2n}} ds\right) \end{aligned} \quad (5)$$

is the potential of s field, whose effective mass is given by $\mu^2(s) = \tilde{V}''(s)$. Thus we have

$$\begin{aligned} \frac{\mu^2(s)}{h^2} & \cong \frac{2+6n - \frac{3}{\sqrt{16\pi n}} \frac{dn}{d\varphi}}{n^2} \\ & \cong \frac{2 - \frac{6}{\epsilon} - \frac{5}{2} \frac{d\ln|\epsilon|}{d\mathcal{N}}}{\left(\frac{1}{\epsilon}\right)^2}, \end{aligned} \quad (6)$$

where the high order terms like $\left(\frac{dn}{d\varphi}\right)^2$ and $\frac{d^2n}{d\varphi^2}$ have been neglected, and in the second line the higher order terms like $\left(\frac{d\ln|\epsilon|}{d\mathcal{N}}\right)^2$ and $\frac{d^2\ln|\epsilon|}{d\mathcal{N}^2}$ also have been neglected. We deduce the second line of Eq.(6) by using the definition of ϵ and \mathcal{N} , and also noting that $\frac{d\epsilon}{d\mathcal{N}}$ is far smaller than ϵ , since $|\frac{1}{n}| \gg |\frac{dn}{(nh)dt}|$, as has been assumed. Thus we see that Eq.(6) is not dependent of n_1 and n_2 , but only the background parameter n or ϵ .

The perturbations will be decomposed into both parts after this rotation is done, one is the curvature perturbation induced by the fluctuation of φ field, and the other is the entropy perturbation induced by the fluctuation of s field. In linear order, as long as the background trajectory remains straight in field space, the entropy perturbation must be decoupled from the curvature perturbation, which actually can be seen in Ref. [30]. When the entropy perturbation is decoupled from the curvature perturbation, the calculation of curvature perturbation is the same as that in Ref. [8], in which only when the increasing mode of metric perturbation before the thermalization may be inherited by the constant model of the curvature perturbation ζ after the thermalization, the spectrum is scale invariant, or the spectrum will be strong blue, whose amplitude is negligible on large scale. In this case the entropy perturbation δs may be calculated as follows. In the momentum space, the equation of entropy perturbation can be given by

$$v_k'' + \left(k^2 - \frac{\beta(\eta)}{\eta^2}\right) v_k = 0, \quad (7)$$

where $\delta s_k \equiv v_k/a$ has been defined and the prime denotes the derivative for the conformal time η , and $\beta(\eta)$ is given by the sum of $\frac{a''}{a}$ and $\mu^2 a^2$, between which is not

subtraction sign as usual, since the fields used here have the reverse sign in their dynamical terms, in which μ^2 is determined by Eq.(6). Note that for $\frac{d\ln|\epsilon|}{d\mathcal{N}} \ll 1$, β is actually near constant for all interesting modes k . Thus Eq.(7) is a Bessel equation and its general solutions are the Hankel functions with the order v given by

$$\begin{aligned} v & = \sqrt{\beta + \frac{1}{4}} \\ & \cong \frac{3 - \frac{2}{\epsilon} + \frac{d\ln|\epsilon|}{d\mathcal{N}}}{2}, \end{aligned} \quad (8)$$

where $\frac{1}{ah} \cong \eta\epsilon(1 - \frac{1}{\epsilon} - \frac{d\ln|\epsilon|}{d\mathcal{N}})$, which has been given in Ref. [31], has been used.

In the regime $k\eta \rightarrow \infty$, all interesting modes are very deep inside the horizon of the parent vacuum, thus Eq.(7) can be reduced to the equation of a simple harmonic oscillator, in which $v_k \sim e^{-ik\eta}/(2k)^{1/2}$. In the superhorizon scale, i.e. $k\eta \rightarrow 0$, in which the modes become unstable and increases, the expansion of Hankel functions to the leading term of k gives

$$v_k \simeq \frac{1}{\sqrt{2k}} (-k\eta)^{\frac{1}{2}-v}, \quad (9)$$

where the phase factor has been neglected. The emergence of island goes with the abrupt change of h , as has been mentioned in last section. Thus it may be expected that the perturbation amplitude of v_k will continue to change after it leaves the horizon, up to the thermalization epoch. This can also be explained as follows. To make the analysis simplified, we assume $|\epsilon|$ is constant. When $k\eta \rightarrow 0$, which corresponds to the super horizon scale, from Eq.(7), we have $v_k'' - \frac{\beta(\eta)}{\eta^2} v_k \simeq 0$. This equation has one increasing solution and one decay solution. The increasing solution is given by $v_k \sim a^{|\epsilon|}$, see Refs. [27, 32] for details. The scale factor a is nearly unchanged, but since $|\epsilon| \gg 1$, the change of v_k has to be significant, thus generally one can not obtain that the $|\delta s_k| = |v_k/a| \sim a^{|\epsilon|}$ is constant, which actually occurs only for the slow roll inflation in which approximately $|\epsilon| \simeq 0$. This result indicates that we should take the value of v_k at the time of thermalization to calculate the amplitude of perturbations. Thus the perturbation spectrum of entropy perturbation is

$$k^{3/2} \mathcal{P}_s^{1/2} = k^{3/2} \left| \frac{v_k(\eta_e)}{a} \right| \sim k^{3/2-v}. \quad (10)$$

Thus the spectrum index is given by

$$n_s - 1 \simeq \frac{2}{\epsilon} - \frac{d\ln|\epsilon|}{d\mathcal{N}}, \quad (11)$$

where Eq.(8) has been used, which means that the spectrum of entropy perturbation generated during the emergence of island is nearly scale invariant, since $|\epsilon| \gg 1$ and $\frac{d\ln|\epsilon|}{d\mathcal{N}} \ll 1$, with a possible tilt determined by the evolution of background. When $|\epsilon|$ is constant, Eq.(11) will be exactly back to that in Ref. [27].

The spectrum of entropy perturbation can be inherited by the curvature perturbation, which can be accomplished by noting that the entropy perturbation sources the curvature perturbation by

$$|\dot{\zeta}| \simeq \frac{2h\dot{\theta}}{\dot{\varphi}} \delta s \quad (12)$$

on large scale [30], where $\theta \equiv \arctg \sqrt{\frac{n_2}{n_1}}$ depicts the motion trajectory of both fields in field space. When θ is a constant, it is a straight line. In this case, $\dot{\theta} = 0$, thus the entropy perturbation will be not decoupled to the curvature perturbation, which also assures the validity of Eq.(7), or there will be some terms such as $\sim \dot{\theta}^2$ and $\sim \dot{\theta}\Phi$. However, if there is a sharp change of field trajectory, $\dot{\theta}$ must be not equal to 0, in this case ζ will inevitably obtain a corresponding change induced by δs by Eq.(12), as has been pointed out and applied in ekpyrotic cosmology [31, 33], see also earlier Refs. [34, 35] and recent studies [36] on the ekpyrotic collapse with multiple fields.

It may be expected that at the split second before the thermalization n_1/n_2 will be not constant any more, since around this epoch both fields will be in the top of their potentials, and thus Eqs.(3) and (4) depicting the upward fluctuation along the potential are generally not valid any more. In this case, the entropy perturbation will be able to source the curvature perturbation. We assume, for a brief analysis, that before the thermalization the motion of φ_2 firstly rapidly stops while the other field φ_1 remains and then will stop moving after several split seconds. Following Ref. [31, 33], this corresponds to a sharp change from initial value $\theta_* = \arctg \sqrt{\frac{n_2}{n_1}}$ to $\theta \simeq 0$. It is this change that leads ζ acquiring a jump induced by the entropy perturbation and thus inherits the nearly scale invariant spectrum of the entropy perturbation. In the rapid transition approximation, one has obtained

$$|\zeta| \simeq \frac{2h\Delta\theta}{\dot{\varphi}} \delta s, \quad (13)$$

where $\Delta\theta \simeq \theta_* = \arctg \sqrt{\frac{n_2}{n_1}}$. The amplitude of entropy perturbation can be calculated at the time of thermalization and given by

$$\frac{k^{3/2}}{\sqrt{2\pi^2}} \left| \frac{v_k(\eta_e)}{a} \right| \simeq |\epsilon| \left(\frac{h_e}{2\pi} \right), \quad (14)$$

where $|\epsilon| \gg 1$ and $\frac{d \ln |\epsilon|}{\mathcal{N}} \ll 1$ have been used. Note that $|h^2/\dot{\varphi}^2| = \frac{4\pi}{|\epsilon|}$, thus we have the amplitude of curvature perturbation

$$\begin{aligned} \mathcal{P}_{(s \rightarrow \zeta)} &= \frac{k^3}{2\pi^2} |\zeta_k|^2 \simeq \left| \frac{h\Delta\theta}{\dot{\varphi}} \right|^2 \cdot \frac{k^3}{2\pi^2} \left| \frac{v_k(\eta_e)}{a} \right|^2 \\ &\simeq (2\Delta\theta)^2 \cdot |\epsilon| \frac{h_e^2}{\pi}. \end{aligned} \quad (15)$$

which is approximately $\mathcal{P}_{(s \rightarrow \zeta)} \simeq |\epsilon| h_e^2$.

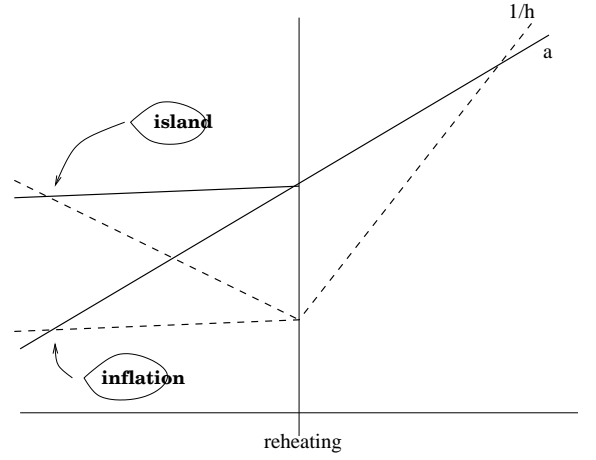


FIG. 3: The figure of the background evolutions of slow roll inflation and island universe model. The black solid line is the scale factor a , while the black dashed line is the horizon radius $1/h$. For slow roll inflation, a is rapidly increased while h is nearly unchanged. For island, a is nearly unchanged while h is rapidly increased. The duality between their scalar spectra is a reflection of that between their background evolutions.

B. The duality of scalar spectra between inflation and island

We can note that Eqs.(11) and (15) can be related to those of the usual slow roll inflation by replacing ϵ as $-\frac{1}{\epsilon}$, which actually exactly gives the spectral index and amplitude of slow roll inflation to the first order of slow roll parameters. This replacement may be regarded as a duality between their scalar spectra, which, in some sense, is a reflection of duality between their background evolutions, i.e. the nearly exponent expansion with $\epsilon \simeq 0$ and the slow expansion with $\epsilon \ll -1$, see Fig.3. In Ref. [27], we showed that this duality is valid for constant $|\epsilon|$, here we find that it is still valid when $|\epsilon|$ changes. This result extends again the studies on the dualities of the primordial density perturbation in Refs. [38, 39], which discussed the cases of $\epsilon > 0$, and [40, 41], in which the case of $\epsilon < 0$ is included.

The duality between the scalar spectra of inflation cosmology and island cosmology indicates that for a slow roll inflation model, we can deduce $\epsilon(\mathcal{N})$ by studying the details of model to calculate its spectrum, however, by the dual relation showed here we also always can write down a dual $\epsilon(\mathcal{N})$ for island universe model, both give same scalar perturbation spectra. For instance, for large field inflation model, we have $\epsilon \sim \frac{1}{\mathcal{N}}$, then in term of duality $\epsilon \rightarrow -\frac{1}{\epsilon}$, we can take $\epsilon \sim -\mathcal{N}$ for island universe, one will find that in this case the scalar spectra of both models will be exactly same. Thus in the level of scalar spectrum, the island universe model is actually degenerated with the slow roll inflation model.

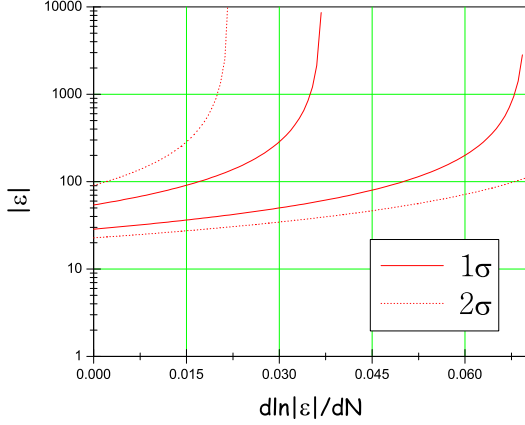


FIG. 4: The region of $|\epsilon|$ with respect to $\frac{d \ln |\epsilon|}{dN}$. The 1σ and 2σ levels are given by WMAP+SDSS. We can see that the region of parameter space consistent with the observations is quite large.

C. The analysis of spectral tilt

The tilt of spectral index given by Eq.(11) is interesting for the observations. When ϵ is constant, the spectrum is slightly red, since $\frac{d \ln |\epsilon|}{dN} = 0$ and $\epsilon \lesssim 0$, which may be matched to the observations well. For instance, taking $|\epsilon| \simeq 50$, we have $n_s \simeq 0.96$, which is well in the region favored by the observation [37]. However, it may be more possible that ϵ is not a constant. In this case, it may be expected that the term $\frac{d \ln |\epsilon|}{dN}$ will significantly affect the spectral index. When $|\epsilon|$ is decreased with the decrease of \mathcal{N} , we have $\frac{d \ln |\epsilon|}{dN} > 0$, which will make the spectrum redder. For instance, taking $|\epsilon| \simeq \mathcal{N}$ at the epoch of $\mathcal{N} \simeq 50$, we have $\frac{d \ln |\epsilon|}{dN} = \frac{1}{\mathcal{N}}$ and thus $n_s \simeq 1 - \frac{3}{\mathcal{N}} \simeq 0.94$. It, of course, is also possible to have a mild red tilt by having a mild running of ϵ with \mathcal{N} , when $|\epsilon|$ is far larger than 10^2 and thus $1/|\epsilon| \rightarrow 0$. For instance, we may take $|\epsilon| \simeq 2.5 \times 10^5 \simeq 10^3 \mathcal{N}^2$ at the epoch of $\mathcal{N} \simeq 50$, and thus may have $\frac{d \ln |\epsilon|}{dN} = \frac{2}{\mathcal{N}}$, and thus $n_s \simeq 0.96$, note that in this case in Eq.(11) $\frac{2}{|\epsilon|} \ll \frac{d \ln |\epsilon|}{dN}$ has been neglected. Thus compared with that of ϵ being constant, in which $|\epsilon|$ is required $\sim 10^2$ by the observations, when ϵ is changed the range of ϵ constrained by the observations may be more flexible, since in this case $|\epsilon|$ may be quite large. We plot Fig.4, in which the region of $|\epsilon|$ with respect to $\frac{d \ln |\epsilon|}{dN}$, consistent with the observation, is given. We can see that when ϵ is constant, i.e. $\frac{d \ln |\epsilon|}{dN} = 0$, in order to match the observation, $|\epsilon|$ must lie between about 20~100, which is a quite constrained region. However, when $\frac{d \ln |\epsilon|}{dN}$ increases, $|\epsilon|$ may be taken as a larger value. From Eq.(15), for $\mathcal{P}_{(s \rightarrow \zeta)} \sim 10^{-10}$, we

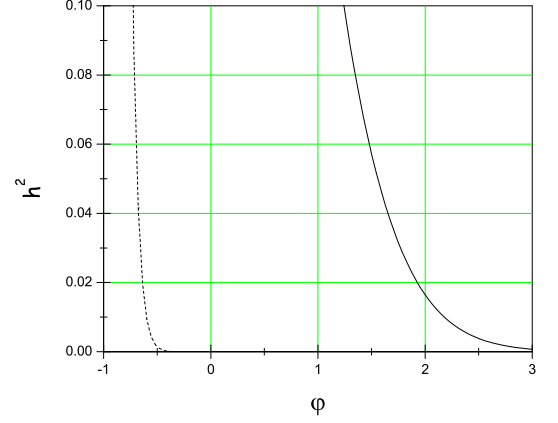


FIG. 5: The change of h^2 with respect to φ , and thus \mathcal{N} . The solid line is that with $|\epsilon| \simeq \mathcal{N}$, and the dashed line is $|\epsilon| \simeq 10^3 \mathcal{N}^2$.

can see that a larger $|\epsilon|$ means a smaller h_e , and thus a lower thermalization temperature, which is actually good for an escaping from the monopole problem afflicting the standard cosmology.

The potential $V(\varphi, s=0)$ of fields can be also obtained when a relation between ϵ and \mathcal{N} is given. We note $\dot{\varphi} = \sqrt{\frac{\hbar}{4\pi}} = \sqrt{\frac{|\epsilon| h^2}{4\pi}}$, and thus have

$$\frac{d\mathcal{N}}{d\varphi} \simeq \frac{\sqrt{|\epsilon|}}{2\sqrt{\pi}}, \quad (16)$$

where $|\epsilon| \gg 1$ has been used, which means that by substituting $\epsilon(\mathcal{N})$, we will obtain a relation between φ and \mathcal{N} . In addition, the function of $n(\mathcal{N})$ can be given by $-\frac{1}{n} \simeq \epsilon - \frac{d\epsilon}{d\mathcal{N}}$. Thus after combining them, we can obtain a function $n(\varphi)$, which then is submitted to Eq.(5) and thus leads to the potential $V(\varphi, s=0)$. For a detail, taking above example $|\epsilon| \simeq \mathcal{N}$, we will have $\mathcal{N} \simeq \frac{\varphi^2}{16\pi}$ by Eq.(16), and then combining it with $\frac{1}{n} \simeq \mathcal{N}$, where $\mathcal{N} \gg 1$ has been used, we can have $\frac{1}{n} \simeq \frac{\varphi^2}{16\pi}$. Thus after substituting it into Eq.(5), we obtain $V(\varphi) \sim \exp(-\frac{\varphi^2}{2})$. The similar steps can be also applied to another example $|\epsilon| \simeq 10^3 \mathcal{N}^2$ mentioned. By combining Eqs.(1), (5) in which $s=0$ is taken, and Friedmann equation, we can straightly obtain the change of h , see Ref. [27] for details. For both above examples, we plot Fig.5 for a comparison between them, in which the calculations are exactly implemented without any approximations. We can see that the case with larger $|\epsilon|$ corresponds to a steeper change of local Hubble parameter, which is an expected result.

In addition, we also can obtain a slightly blue spectrum by requiring $\frac{d \ln |\epsilon|}{dN} < 0$ and $\left| \frac{d \ln |\epsilon|}{dN} \right| > \frac{2}{|\epsilon|}$. This may be

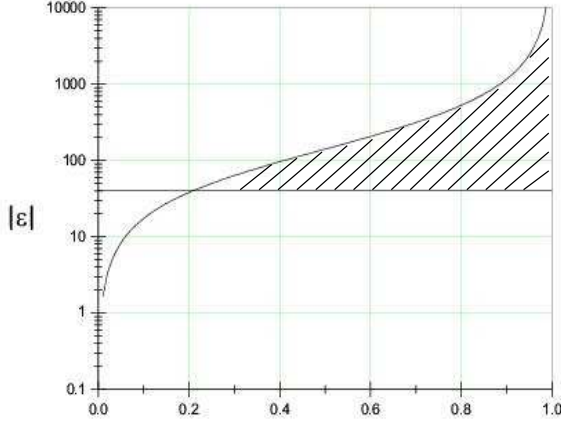


FIG. 6: The region with tilt lines is that consistent with the non Gaussianity observation, in which the horizon axis is n_1/n_2 . $|\epsilon|$ is assumed as constant, and the upper limit is determined by $f_{\text{NL}} < 100$ and the lower limit is given by $\frac{2}{|\epsilon|} \simeq 0.05$.

implemented e.g. by taking $|\epsilon| \simeq \frac{10^6}{N}$ at the epoch of $N \simeq 50$, which leads to $n_s - 1 \simeq 0.01$ given by Eq.(11). Thus in principle we could have any tilt required by the observations in such an island universe thermalized in the landscape.

D. The non-Gaussianity of curvature perturbation

Then we will simply estimate the non-Gaussianity of this curvature perturbation. Here the curvature perturbation is induced by the entropy perturbation from δs through Eq.(13), thus in principle the non-Gaussianity has two sources, one is the cubic interaction terms of s field, the other is the nonlinear relation between δs and ζ , noting that Eq.(13) is only the result in linear approximation. Here we will estimate the contribution to the non-Gaussianity from the cubic interaction terms of s field. To simplicity, we will assume that n_1 and n_2 and thus n are constant. In this case, the interaction Hamiltonian is $\mathcal{H}_{\text{int}} = \alpha \frac{(\delta s)^3}{\sqrt{n}(-t_e)^2}$, where $\alpha = (\sqrt{\frac{n_1}{n_2}} - \sqrt{\frac{n_2}{n_1}}) \frac{\sqrt{16\pi}}{3}$ has been set, which can be obtained by expanding the potential $V(\varphi, s)$ in Eq.(5) and then taking the cubic part of s , in which Eqs.(1) and (5) and Friedmann equation have been used. Following [42], and also recent [43], the 3-point function of δs is given by

$$\begin{aligned} & \langle \delta s_{\vec{k}_1} \delta s_{\vec{k}_2} \delta s_{\vec{k}_3} \rangle \\ &= -i \int_{-\infty}^{t_e} \langle [|\delta s_{\vec{k}_1} \delta s_{\vec{k}_2} \delta s_{\vec{k}_3}, \mathcal{H}_{\text{int}}(\lambda)] \rangle d\lambda + \text{c.c.} \\ &\simeq (2\pi)^3 \frac{\delta \left(\sum_i \vec{k} \right) \sum_i k_i^3}{\prod_i k_i^3} \cdot \frac{\alpha}{\sqrt{2^3 n}(-t_e)} \mathcal{P}_s^{3/2}, \end{aligned} \quad (17)$$

which is calculated at the time of thermalization, where the second line is obtained by only reserving the leading order contribution for small t_e , and k_i is the amplitude of \vec{k}_i and $\mathcal{P}_s \simeq \frac{1}{2(-t_e)^2}$ is given by Eq.(10), since $a\eta_e \simeq t_e$ for $\epsilon \ll -1$ and also $v \simeq 3/2$.

Thus with Eq.(13), in super Hubble scale, the 3-point function $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle$ of curvature perturbation induced by δs can be written as

$$\pm (2\pi)^3 \frac{\delta \left(\sum_i \vec{k} \right) \sum_i k_i^3}{\prod_i k_i^3} \cdot \frac{\alpha}{\sqrt{2^3 n}(-t_e)} \mathcal{P}_{(s \rightarrow \zeta)}^{3/2}, \quad (18)$$

where \pm appears since generally $\zeta \sim \pm \delta s$ and $\mathcal{P}_{(s \rightarrow \zeta)}$ is given by Eq.(15). The level of non-Gaussianity is usually expressed in term of parameter f_{NL} [44, 45] defined as $\zeta(x) = \zeta_g(x) - \frac{3}{5} f_{\text{NL}} \zeta_g^2(x)$. Thus combining it with Eq.(18), we have

$$\begin{aligned} f_{\text{NL}} &\simeq \mp \frac{5\sqrt{2}}{24} \frac{\alpha}{\sqrt{n}(-t_e)} \frac{1}{\sqrt{\mathcal{P}_{(s \rightarrow \zeta)}}} \\ &\simeq \mp \frac{5\sqrt{\pi}}{24\sqrt{2}} \cdot \frac{\alpha}{\Delta\theta} |\epsilon|, \end{aligned} \quad (19)$$

where Eq.(15), and also $n \equiv \frac{1}{|\epsilon|}$ when n is constant, have been used. Eq.(19) shows that the non-Gaussianity is proportional to $|\epsilon|$, which is similar with that of usual slow roll inflation model in which since $\epsilon \ll 1$ the non-Gaussianity is generally very small. However, in island cosmology, since $|\epsilon| \gg 1$, the non-Gaussianity is generally quite large. This can also occur samely in ekpyrotic cosmology [43, 46, 47], in which $\epsilon \gg 1$ while here $\epsilon \ll -1$, see also recent different study [48]. However, note also that here α is actually dependent of n_2/n_1 . When $n_2 = n_1$, we can obtain $\alpha = 0$. Thus it is also possible to make the level of non-Gaussianity be quite small by adjusting n_1 and n_2 , i.e. the ratio between both fields contributing the background, which is actually determined by their potentials. The observation gives $-36 < f_{\text{NL}} < 100$ [37]. Thus an estimate of the upper limit of ϵ for different n_2/n_1 may be obtained by noting $\Delta\theta \simeq \arctan \sqrt{\frac{n_2}{n_1}}$, which is plotted in Fig.6, in which we take $f_{\text{NL}} < 100$. We can see that when $n_2/n_1 \rightarrow 1$, ϵ may be quite liberal, however, in general case there is an upper limit for ϵ . For instance, taking $n_2/n_1 \simeq 0.5$, we have $|\epsilon| \lesssim 150$. Thus dependent of different evolutions, the island universe can have large or small non-Gaussianity, which makes it safely lie within the observational bound. Here we do not consider the contribution of the nonlinear relation between δs and ζ to the non-Gaussianity, however, this contribution is actually the same order as that given by the cubic interaction terms of s field and thus hardly can affect our rough estimate made here. We will back the detailed study of non-Gaussianity of island universe in the future, noting the significant detection of non Gaussianity [49].

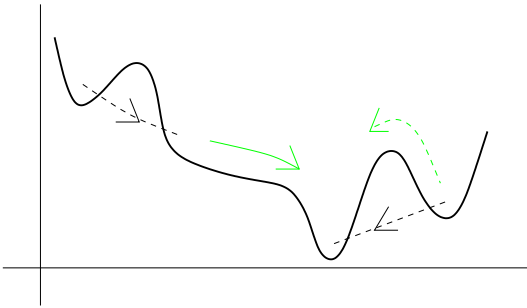


FIG. 7: In the eternal inflation led by the landscape, the observable universes may be some of many thermalized regions spawned within the eternally inflating background. They appear either by a slow roll inflation after the nucleation of bubbles, which is generally induced by the CDL instanton, like that given by the black dashed line followed by the green solid line, or by a straightly thermalization in new vacua without the slow roll inflation, which may be induced by the HM instanton or others, like that given by the green dashed line. In the meantime it can also be expected that there are also lots of regions, which are empty and not thermalized, like that given by the black dashed line in right side.

IV. DISCUSSION

In this paper, we have illustrated and calculated the spectrum of primordial perturbation of the island universe in the landscape. The landscape can be depicted as the space of a set of fields. The calculations implemented here closely capture this character of landscape, since in this case the emergence of island will inevitably involve the fluctuations of many fields, and thus the entropy perturbation can be generated, which may induce the curvature perturbation under certain condition. We showed a detailed results of the spectrum of curvature perturbation induced by the entropy perturbation and discussed that the parameter space required by the observations should correspond to what change of local Hubble parameter during the emergence of an island. We find that in general case the results obtained can be well related to those of slow roll inflation by a simple duality. In addition, we also simply estimate the non-Gaussianity of perturbation. The results shown here indicates that the island universe in the landscape can be consistent with our real world.

Thus given the landscape, the observable universes may be some of many thermalized regions spawned within the eternally inflating background, which appear either by a slow roll inflation after the nucleation of bubbles, followed by the reheating, or by a straightly thermalization in new vacua without the slow roll inflation, like islands, see Fig.7 for the illustration. Thus it is interesting to ask how we know whether we live in an emergently thermalized island or in a reheating region after

slow roll inflation inside bubble. The island universe generally has a large non-Gaussianity, which can be distinguished by coming observations, as has been shown here. However, a large non-Gaussianity can be also achieved in some special inflation models. In addition, it has been shown that in the island the tensor amplitude is negligible on large scale [6, 8], while there exists a large class of inflation models, such as large field inflation model, with moderate amplitude of tensor perturbation, see e.g. Ref. [50] for the various inflation models. Thus it seems that the detection of a stochastic tensor perturbation will be consistent with the inflation model, while rule out the possibility that an straightly thermalized region is regarded as our real world. However, low tensor amplitude on large scale is also not conflicted with the inflation model, e.g. some small field inflation models. Thus in this case other distinguishabilities need to be considered. The bubble after the nucleation described by the CDL instanton is generally negatively curved, and thus the corresponding universe is an open universe, while the island generated by the upward fluctuation may be closed. Thus in this sense it seems that the curvature measurement of our universe will be significant to make clear where we live in. However, we still need to look for more distinct signatures for island universe, which will be backed to in intending works.

The emergence of island here is depicted as an upward fluctuation with the null energy condition violation, which might be interesting for the discussions of the initial conditions of observable universe, see Refs. [51] and also recent [52, 53], since it might be closely relevant to the solution of above issue. The emergence probability of island can be approximately given by that of the HM instanton, which is actually exponentially suppressed. However, in the bubble nucleated by the CDL instanton, in order to have an universe like ours, the slow roll inflation with enough period is generally required. This only can be implemented by having a potential with a long plain above its minimum, which obviously means a fine tuning, since the regions with such potentials are generally expected to be quite rare in a random landscape. While the island of observable universe can actually emerge for any potential, independent of whether the potential has a long plain, as long as we can wait. Thus in principle the island of observable universe can exist in any corner of landscape. This in some sense brings us an interesting expectation that we might live in a straightly thermalized “island” in the landscape.

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